Math Virtual Learning

Probability and Statistics

**April 24, 2020**

Probability and Statistics
Lesson: April 24, 2020

Objective/Learning Target:

Students will be able to convert a percentage into its correlating Z-Score to then find the data point associated with that Percentage

Let's Get Started!

Use the Z Score to Percentage Conversion Chart to answer the following questions

Research was done on a local chicken farm to determine the number of eggs produced daily. It was found that on average, 203 eggs were produced each day. The standard deviation in this study was 14. (FYI - The data is normal)
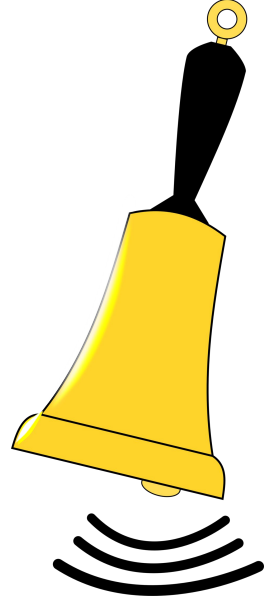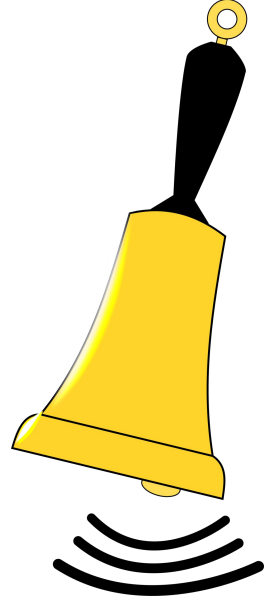
What is the z-score for 178 eggs?

What percentage of days does the farm produce 178 eggs or less?

What percentage of days does the farm produce more than 178 eggs?

Let's Get Started!

Use the [Z Score to Percentage](#) Conversion Chart to answer the following questions

Research was done on a local chicken farm to determine the number of eggs produced daily. It was found that on average, 203 eggs were produced each day. The standard deviation in this study was 14.

What is the z-score for 178 eggs?

$$\frac{178 - 203}{14} = -\frac{25}{14} = -1.79$$

What percentage of days does the farm produce 178 eggs or less?
Look on the chart where -1.7 and 0.09 intersect and it says 0.0367 = 3.67%

What percentage of days does the farm produce more than 178 eggs?
100 - 3.67 = 96.33%

What we already know...

Now that we know how to calculate a Z-Score using the formula

AND we know how to look up the percentage below from the chart

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545

Today we will learn Percent to Z-Score...

Now that you can answer questions about percentages using a Z-Score to Percent Chart, we will look at taking a percentage and turn it back into a Z-Score to then find the data point.

Basically working the process we learned yesterday, only…

BACKWARDS

Just like yesterday, you will need this chart again to do this assignment…

Z-Score to Percent Chart

Let's start with an example...

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

On one subtest, the raw scores have a mean of 35 and a standard deviation of 6.

Assuming these raw scores form a normal distribution:

a) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

So this time you are trying to figure out what test score is at the 65th percentile.

1. **Since we know the %, we have to find it on the Z-Score to Percent Chart *(it will be an approximation)***

2. **Next we use the column and row to get the Z-Score associated with that %**

3. **Next we put that Z-Score into the formula**

4. **Lastly, we solve for X**

Like this...

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

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Assuming these raw scores form a normal distribution:

a) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

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0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

STEP 1:
Search the positive side of the chart *(because 65% is higher than 50% so it will be on the right side of the mean)* until you find decimals that are APPROXIMATELY .65

I found two: **.6480 and .6517**

Now you have to determine which one is closer to .65

.6480 is .0020 away on the low side
.6517 is .0017 away on the high side

So .6517 is closest to .65

Like this...

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

On one subtest, the raw scores have a mean of 35 and a standard deviation of 6.

Assuming these raw scores form a normal distribution:

a) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

STEP 2:

If you look at the chart and you follow the yellow line to the left and the green line to the top, you will see that .6517 correlates to the Z-Score of **0.3 (yellow) and 0.09 (green)**

Put those together and you get a final Z-Score of 0.39

Like this...

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

On one subtest, the raw scores have a **mean of 35** and a **standard deviation of 6.**

Assuming these raw scores form a normal distribution:

a) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

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CONCLUSION:
37.34 is the approximate score that represents the 65th percentile.
Approximately 65% of the people scored LOWER than a 37.34 on this subtest.

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Score

Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD

STEP 3 & 4:

Take the Z-Score (0.39) and plug it into the Z-Score formula and solve for X

Mean: 35   S.D.: 6

$$0.39 = \frac{x - 35}{6}$$

2.34 = x - 35   **(multiply both sides by 6)**
x = 37.34   **(add 35 to both sides)**

Your turn… (1 of 2 practice problems)

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

On one subtest, the raw scores have a **mean of 35** and a **standard deviation of 6.**

Assuming these raw scores form a normal distribution:

b) What number represents the 90th percentile?

Your turn… (1 of 2 practice problems) ANSWER

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

On one subtest, the raw scores have a **mean of 35** and a **standard deviation of 6.**

Assuming these raw scores form a normal distribution:

b) What number represents the 90th percentile?

Step 1: Find 90% on the positive side of the chart *(approximately)*: **.8997 is closest to .90**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
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0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
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0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
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1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177

Your turn… (1 of 2 practice problems) ANSWER

The Welcher Adult Intelligence Test Scale is composed of a number of subtests.

On one subtest, the raw scores have a **mean of 35** and a **standard deviation of 6.**

Assuming these raw scores form a normal distribution:

b) What number represents the 90th percentile?

Step 2:

.8997 corresponds to a Z-Score of 1.28

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
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1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177

Your turn… (1 of 2 practice problems) ANSWER

Step 3: Put your answer into the Z-Score Formula

Score

Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD

$$1.28 = \frac{x - 35}{6}$$

Step 4: Solve for x

7.68 = x - 35   **(multiply both sides by 6)**

x = 42.68   **(add 35 to both sides)**

CONCLUSION
A score of approximately 42.68 represents the 90th percentile. 90% of the people scored LESS than 42.68 on this subtest.

Your turn… (2 of 2 practice problems)

Scores on the SAT form a normal distribution with mean of 500 and standard deviation of 100 .

a) What is the minimum score necessary to be at the 3rd percentile?

Your turn… (1 of 2 practice problems) ANSWER

Scores on the SAT form a normal distribution with mean of 500 and standard deviation of 100 .

a) What is the minimum score necessary to be at the 3rd percentile?

Step 1: Find 3% on the negative side of the chart *(approximately)*: **.0301 is closest to .03**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367

Your turn… (2 of 2 practice problems) **ANSWER**

Step 2:

.0301 corresponds to a Z-Score of -1.88

Because the percentage of 0.0301 lines up with -1.8 and 0.08.

So the z-score is -1.88

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367

Your turn… (2 of 2 practice problems) **ANSWER**

Step 3: Put your answer into the Z-Score Formula

Score → Mean →

$$Z = \frac{x - \mu}{\sigma}$$

↖ SD

$$-1.88 = \frac{x - 500}{100}$$

Step 4: Solve for x

-188 = x - 500   **(multiply both sides by 100)**

x = 312   **(add 500 to both sides)**

> **You would need a minimum of 312 on the SAT to score in AT LEAST the 3%**